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Babylonian Shadow-Length Schemes: Between Mathematics and Astronomy

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Abstract

A simple mathematical scheme to represent the variation in the length of the shadow cast by a vertical gnomon at different times of day and in different months of the year is presented in the early astronomical compendium MUL.APIN. A small number of texts composed in the Late Babylonian period investigate and expand this scheme. These texts have previously been studied and understood as part of Babylonian astronomy. In this article, I suggest that two of these later texts can be better understood as mathematical texts. As such they provide evidence for the influence of astronomy on Late Babylonian mathematics, either or both as the context for simple mathematical problems and/or as a topic of mathematical investigation.

Key-words: astronomy; gnomon; scholarly interaction; mathematics; shadows

Esquemas babilónicos de longitud de la sombra: entre las matemáticas y la astronomía

Resumen

Un simple esquema matemático para representar la variación en la longitud de la sombra proyectada por un gnomon vertical en distintos momentos del día y en diversos meses del año se presenta en el

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compendio astronómico MUL.APIN. Un pequeño número de textos compuestos en el período neobabilónico investigan y expanden este esquema. Estos textos han sido previamente estudiados y entendidos como parte de la astronomía babilónica. En este artículo sugiero que dos de estos textos tardíos pueden ser mejor entendidos como textos matemáticos. Como tales, informan sobre la influencia de la astronomía en las matemáticas neobabilónicas, ya sea como contexto para simples problemas matemáticos y/o como argumento de investigación matemática.
Palabras clave: astronomía; gnomon; interacción científica; matemáticas; sombra

1 Introduction

Embedded within Babylonian astronomical practice is a body of mathematical knowledge and techniques of calculation which draw upon, and sometimes expand upon, existing mathematical traditions, including systems of metrology, simple arithmetic operations, and a fluency in working with tables and the functions which underlie their construction. This reliance of astronomy upon mathematics extends to every type of astronomy including the observation of celestial phenomena, the description of the behaviour of the sun, moon, and other bodies in the sky using mathematical schemes, and the prediction of future astronomical phenomena from past observations, as well as, most obviously, the various systems of mathematical astronomy which calculate astronomical phenomena using mathematical functions without the necessity of regular empirical input[1]. The influence of astronomy on Babylonian mathematics is harder to pin down, however. The so-called Hilprecht Text, a text known from one Middle Assyrian and two Neo-Assyrian copies (Oelsner 2005–06), uses distances between stars as the context for a mathematical problem (Rochberg-Halton 1983), and Huber (1957: 279–281) has suggested that one problem on AO 6484, a mathematical problem text from Seleucid Uruk, might have had an application in the planetary systems of mathematical astronomy, but these are isolated examples found among a very large corpus of mathematical

[1]For a comparison of mathematical methods and terminology found in Old Babylonian mathematics, Late Babylonian mathematics, and Late Babylonian mathematical astronomy, see Ossendrijver (2012: 26–27). For a detailed discussion of one particular example, the use of calculations using a trapezoid to compute Jupiter's motion and its mathematical context, see Ossendrijver (2018).

texts. Further direct evidence for astronomy providing either the setting for mathematical problems or as the context for developments within mathematics is scarce.

In this paper, I point to another possible case of the presence of astronomy within a mathematical context. Several cuneiform texts contain schemes relating the length of shadow cast by a vertical gnomon to the time of day on specific dates during the year (Steele 2013). It has long been recognized that the foundation of these shadow-length schemes is a purely mathematical reciprocal relationship between the length of the shadow and the time after sunrise (van der Waerden 1951: 34; Neugebauer 1975: 544–545; Steele 2013: 9). As I will show, the properties of this mathematical relationship were explored in both astronomical and mathematical texts.

2 Late Babylonian mathematical and astronomical texts

Roughly five thousand cuneiform tablets from first millennium BC Babylonia contain astronomical or astrological texts. These texts include several large groups of tablets that are the product of ongoing astronomical practice, such as the so-called Astronomical Diaries, which contain records of astronomical observations and other material for half a year, the Goal-Year Texts, which contain collections of observations to be used in the process of predicting future astronomical phenomena, the Almanacs and Normal Star Almanacs, which contain the resulting predictions for a given year, and the Synodic Tables of mathematical astronomy which contain calculated astronomical data produced by implementing purely mathematical algorithms without regular empirical input; reference texts such as star lists and procedure texts explaining how to make astronomical predictions; and a variety of other texts, many of which appear to be one-off compositions. In addition, a significant number of tablets contain astrological texts, including horoscopes, which contain astronomical data for the birth of an individual, and texts stating astrological associations between, for example, the signs of the zodiac and medical ingredients, geographical regions, personal characteristics, and the potential length of an individual's life. Finally, we find copies of earlier standard works of astronomy and astrology, such as the astronomical compendium MUL.APIN and the collection of celestial omens *Enūma Anu Enlil*, and commentaries and other new compositions based upon these works.

The first millennium BC corpus of mathematical texts is somewhat smaller than both the contemporary astronomical and astrological corpus and the corpus of mathematical texts from the second millennium BC. The mathematical corpus includes school texts which contain basic mathematical and metrological material, metrological lists and tables, and what I will call "advanced scholarly mathematics", namely texts containing mathematical problems and mathematical tables that are the result of complex, and often seemingly pointless, calculations such as lists of the ninth powers of long sexagesimal numbers[2].

Tablets containing advanced scholarly mathematics and tablets containing astronomical texts have been found together in several Late Babylonian scholarly archives. For example, mathematical and astronomical tablets have been recovered from both stages of occupation of the so-called 'House of the *āšipu*s' in Uruk. This house was the home of two successive families of *āšipu*s: the Šangî-Ninurta family in the late fifth and early fourth century BC and then, following a period of abandonment, the Ekur-zakir family from the mid fourth to the late third century BC (Robson 2008: 224–240; Clancier 2009: 47–62; Proust and Steele 2019). Members of both families owned and copied a wide range of scholarly tablets including several astronomical, astrological, and mathematical tablets (Proust 2019; Steele 2019). Similarly, mathematical and astronomical tablets were found alongside one-another in the archives of the Bīt Rēš temple in Hellenistic Uruk (Ossendrijver 2019). Thus, at least in the second half of the first millennium BC, astronomy and mathematics were practiced by the same individuals and in the same contexts. The tablets containing material

[2]Babylonian astronomy and mathematics made extensive use of the sexagesimal place-value number system. In our decimal system, we have digits from 0 to 9. The value of a digit depends upon its place in a number. Thus the digit 1 can be used to write the number one, or when followed by a second digit such as 3 it has the meaning of one ten + three units or thirteen. Large numbers can be written by adding further digits to the left. Decimal fractions can be indicated by adding digits to the right of the decimal point marker. The sexagesimal place-value system operates on the same principle but with 'digits' from 0 to 59. It is conventional among those working on Babylonian astronomy to transcribe sexagesimal numbers using commas to separate 'digits'. In the Babylonian sexagesimal system, however, the magnitude of a number is usually unspecified and there is no equivalent of the decimal point. Thus any number can be multiplied by any positive or negative factor of 60. For example, the sexagesimal number 1,30 can be understood as one sixty plus thirty units (= 90), or one three-thousand-six-hundred plus 30 sixties (= 5400), or one unit plus thirty sixtieths (= 1 30/60th = 1.5), etc. If the absolute value of a sexagesimal number is know from its context, this is indicated in translations (but not transliterations) by a semicolon as the equivalent of the decimal point.

relating to the length of the shadow cast by a gnomon, which are the focus of this paper, were found in these same contexts and it therefore seems certain that they were written by the same individuals who were writing astronomical and mathematical texts.

Although astronomy and mathematics were practiced by the same scholars, these scholars clearly distinguished these activities, following different conventions for how to present their scholarship depending upon the genre in which they were writing. One of the main ways of presenting mathematics, for example, was through the form of what are referred to as "problem texts" in modern scholarship. These texts, which have a long history stretching back to the early second millennium BC, typically start with some data and a question to which the answer is then given. Consider the following example from a collection of problems on a tablet owned by Anu-abu-utēr, who was active in the early second century BC[3]:

> *tam-ḫír-tu*$_4$ *šá* TA 1 GAM 1: 1 EN 10 GAM 10: 1,40 *ki-i* EN ŠID-*tú* 1 GAM 20 : [1/3] DU-*ma* 20 : 10 GAM 40 : 2-TA ŠU.MIN.MEŠ DU-*ma* 6,40 : 6,40 *ù* 20 7 7 GAM 55 DU-*ma* 6,25 : 6,25 ŠID-*tú*
>
> Squares which are from 1 by 1 (is): 1 to 10 by 10 (is): 1,40. What is the count? 1 by 20 (is): [1/3] and it is 0,20. 10 by 40 (is): 2/3 and it is 6,40. 6,40 and 0;20 (together are) 7. 7 by 55 and it is 6,25. 6,25 is the count.

In this problem, the reader is presented with a situation, a series of successive squares which are from 1 by 1 to 10 by 10, and asked 'what is the count?', meaning what is the sum of these squares. We are then presented with a calculation in order to find the result, which is correctly given as 6,25. Consider now the following extract from a contemporary astronomical procedure text[4]:

> ⌜MÚL.BABBAR⌝ TA 9 ALLA EN 9 GÍR.TAB 30 TAB *šá al-la* 9 GÍR.TAB DIRI A.⌜RÁ⌝ 1,7,[30 DU] ⌜TA⌝ 9 GÍR.TAB EN 2 MÁŠ 33,⌜45⌝ TAB *šá al-la* 2 MÁŠ DIRI A.RÁ 1,⌜4⌝ DU TA 2 MÁŠ EN 17 MÚL.MÚL 36 TAB ⌜*šá al-la*⌝ 17 MÚL.MÚL DIRI

[3]AO 6484 Obv. 3–5. See Neugebauer (1935–7: 96–107) for a detailed discussion of the problems on this tablet.

[4]BM 33869 Obv. 1–4. See Ossendrijver (2012: 288–290) for a full edition and study of this tablet.

A.RÁ 56,15 DU TA 17 MÚL.MÚL EN 9 ⌜ALLA⌝ 33,⌜45⌝ [TAB] ⌜*šá al*⌝-*la* 9 ALLA DIRI A.RÁ 53,20 DU

Jupiter: From 9 Cancer to 9 Scorpio add 30. That which exceeds 9 Scorpio multiply by 1;7,30. From 9 Scorpio to 2 Capricorn add 33;45. That which exceeds 2 Capricorn multiply by 1;4. From 2 Capricorn to 17 Taurus add 36. That which exceeds 17 Taurus multiply by 0;56,15. From 17 Taurus to 9 Cancer add 33;45. That which exceeds 9 Cancer multiply by 0;53,20.

This passage presents the necessary information to calculate the position of Jupiter in the zodiac at one of its synodic phenomena (first visibility, first station, acronychal rising, second station, or last visibility) from its position in the zodiac the previous time it exhibited the same phenomenon. In contrast to the mathematical problem text, rather than a question being posed, a calculation to solve the question presented, and a statement of the answer, in this astronomical procedure text we get a set of instructions that allow the planet's position the next time it exhibits a particular synodic phenomenon to be calculated if we know its position at the same phenomenon. Astronomical procedure texts are written in the form of instructions. Similar to other instructional texts, such as the texts describing how to make glass, the procedures are presented as a series of second-person instructions (implicitly in the example just quoted, but explicitly in some other astronomical procedure texts) and sometimes they are introduced by the phrase "in order for you to make x" (x *ana* DÙ-*ka*), where x is what the reader is being told what to compute[5].

Thus, we have a clear difference in style between mathematical problem texts and astronomical procedure texts. In the former, we have specific questions introduced by the word *mīnû* "what", written either syllabically or using the logographs EN.NAM, EN, or, very occasionally in the late period, ḪÉ.EN, usually followed by a concrete numerical calculation, which serves to illustrate a mathematical problem. In astronomical texts, however, we find generalized procedures introduced by the phrase "in order for you to make". The term *mīnû* only appears within (not introducing) procedures as part of the way that a division is expressed. For example, in a procedure text concerning Mars we find the following[6]:

[5]A similar phrase "you, in your making" (*at-ta i-na e-pí-ši-ka*) appears in some mathematical texts introducing the solution to a problem, but never, as far as I know, in introducing the problem itself.

[6]BM 34676 Rev. 33'– 34 (Ossendrijver 2012: 232–233).

> *šá* AN 4,44 MU.MEŠ ⌜2,13 x⌝ IGI.MEŠ 2,31 BAL.MEŠ 15,6 KI DU *mi-nu-ú* A.RÁ 2,13 IGI.MEŠ *lu*-DU-*ma* *lu*-⌜*ú*⌝ 15,6: 6,48,43,18,30 GAM 2,13 IGI.MEŠ DU-*ma* 15,6: 6 *tu*-⌜*ub*$^{?}$⌝-*al* 48,43,18,40 *ana* MURUB$_4$-*ú* GAR-*an*
>
> Concerning Mars: 4,44 years, 2,13 ... appearances, 2,31 revolutions, the position proceeds by 15,6,0. What should I multiply by 2,13 appearances so that it is 15,6,0?: You multiply 6,48;4318,30 by 2,13 appearances, it is 15,6,0.You subtract 6,0 (and) put down 48;43,18,0 for the middle one.

The procedure is concerned with finding the mean synodic arc between two consecutive occurrences of one of the phenomena of Mars from the fact that in 4,44 (= 284) years, Mars exhibits its synodic phenomena 2,13 (= 133) times and has made 2,31 (= 151) circuits around the zodiac. The mean synodic arc is simply the total distance moved by Mars, which is given by 2,31 multiplied by 360 UŠ (where 1 UŠ = 1 degree) which is 15,6,0 (= 54360), divided by the number of occurrences of the phenomena, 2,13. Thus, the mean synodic arc is equal to 15,6,0 divided by 2,13. It is in performing this division where we encounter the word *mīnû* "what": we are asked "what" should be multiplied by 2,13 to give 15,6,0 and told that the answer is 6,48;43,18,30. This value is therefore the mean distances traveled by Mars between successive occurrences of the phenomenon. Finally, we subtract 6,0 (= 360) from the result to eliminate complete circuits of the zodiac. Note that within this procedure, the term *mīnû* "what" was not used to set up the overall question – there is no statement asking "what is the mean synodic arc?" – instead it is only used as part of a formulaic phrase when presenting the division of one number by another.

3 The basis of Late Babylonian shadow-length schemes

Underlying all Late Babylonian discussions of the length of the shadow cast by a gnomon is the shadow-length scheme presented in the early astronomical compendium MUL.APIN. This widely copied text, which was composed sometime in the late second or the early first millennium BC, contains a collection of star lists and mathematical schemes describing astronomical phenomena including the variation in the length of day and night across the year, the duration of visibility of the moon on each day of a month, as well as descriptions of the synodic cycles of the planets, intercalation rules, and a short collection of celestial omens (Hunger and Steele 2019). MUL.APIN was a well know and widely read composition during the Late Babylonian period and formed the basis for a distinct type of astronomy which I name "schematic astronomy" and which flourished alongside the observational and predictive astronomical traditions of the late period (Steele 2021).

The shadow-length scheme in MUL.APIN is divided into four sections, each of which contains statements of the length of day and night and the time after sunrise at which the shadow cast by a vertical gnomon of one cubit in height reaches specified lengths. The four sections concern the 15th of Months I, IV, VII, and X. These dates correspond to the dates of the solstices and equinoxes in the schematic 360-day calendar in which it is assumed that the year contains 12 months each of exactly 30-days. This schematic calendar is used throughout MUL.APIN and the schematic astronomy tradition. Immediately following the four sections containing the shadow length data, the next two lines present a short supplementary procedure which indicates how the scheme can be expanded to the other months of the schematic calendar.

Let us examine the first section, which presents data for the 15th of Month I, the date of the spring equinox in the schematic calendar[7]:

> DIŠ *ina* $^{\text{iti}}$BÁR UD.15.KAM 3 MA.NA EN.NUN u_4-*mi* 3 MA.NA EN.NUN GE$_6$ 1 *ina* 1 KÙŠ GIŠ.MI 2 1/2 DANNA u_4-*mu* 2 *ina* 1 KÙŠ GIŠ.MI 1 DANNA 7 UŠ 30 NINDA u_4-*mu* 3 *ina* 1 KÙŠ GIŠ.MI 2/3 DANNA 5 UŠ u_4-*mu*
>
> ¶ Month I, the 15th day, 3 mina is the watch of the day, 3 *mina* is the watch of the night. 1 cubit of shadow (at) 2 1/2 *bēru* of daytime. 2 cubits of shadow (at) 1 *bēru* 7 UŠ 30 NINDA of daytime. 3 cubits of shadow (at) 2/3 *bēru* 5 UŠ of daytime.

The section begins with the statement that on the 15th of Month I, day and night both last for 3 *mina*. A *mina* is a unit of weight and is sometimes used to express the duration of day or night, the implication being that this is the weight of water flowing through a waterclock in the corresponding length of time. The following lines give times using the units *bēru*, UŠ and

[7]MUL.APIN II ii 21–24 (Hunger and Steele 2019: 152–153).

NINDA, where there are 30 UŠ in a *bēru* and 60 NINDA in an UŠ. These are standard units of fixed-length time used in Babylonia. A full day (i.e. daytime + night) lasts for 12 *bēru* or 360 UŠ. This being the case, 1 *mina* must correspond to 2 *bēru* or 60 UŠ of time in the first line here. From other parts of MUL.APIN, we know that the variation in the length of daylight was expressed as a simple zigzag function with a period of twelve months and with maximum and minimum values at the solstices of 4 *mina* or 240 UŠ and 2 *mina* or 120 UŠ[8]. The next three lines give the time after sunrise at which the shadow is 1 cubit, 2 cubits, and 3 cubits in length respectively. These time intervals after sunrise are given using the time units *bēru* and UŠ. For simplicity, we can convert all of these time intervals into the unit UŠ, as I have done in table 1. It will be apparent from this table that the length of the shadow multiplied by the time after sunrise is always equal to 1,15 (= 75).

Shadow length	Time interval as written in text	Time interval in UŠ
1 cubit	2 1/2 *bēru*	1,15
2 cubits	1 *bēru* 7 UŠ 30 NINDA	37;30
3 cubits	2/3 *bēru* 5 UŠ	25

Table 1: The shadow length data given in MUL.APIN for the spring equinox.

The next three sections contain similar entries for the dates of the summer solstice (15th of Month IV), the autumnal equinox (15th of Month VII), and the winter solstice (15th of Month X). As we would expect, the data for the autumnal equinox is identical to that of the spring equinox just discussed. For the two solstices, data for shadow lengths ranging from 1 to 10 cubits in 1 cubit intervals are given, with the exception of the case of 7 cubits which is omitted for reasons which will become clear. In each case, the length of shadow multiplied by the time after sunset is equal to a constant. That constant is 1,0 (= 60) for the summer solstice, 1,15 (= 75) again for the autumnal equinox, and 1,30 (= 90) for the winter solstice. No data is given for 7 cubits because 7 does not have a terminating reciprocal in base 60, and so the time when the shadow reaches 7 cubits cannot be expressed precisely using the units *bēru*, UŠ and NINDA.

The short procedure in the lines following the scheme allows it to be extended to all months of the year and to all shadow lengths between 1

[8]Note that this ratio of longest to shortest day of 240 UŠ : 120 UŠ = 2 : 1 is very inaccurate for the latitude of Babylon.

and 10 cubits except, as mentioned, for 7 cubits. The extended scheme is presented in table 2. It can be seen from inspecting this table that the shadow length scheme is founded upon two basic mathematical rules. Entries within a column, i.e. for the same month, follow the rule that the product of the shadow length and the time after sunrise is equal to a constant *c*. Entries along a row, i.e. for the same shadow length, follow simple zigzag functions with a period of 12 months.

	Month I	Month II	Month III	Month IV	Month V	Month VI	Month VII	Month VIII	Month IX	Month X	Month XI	Month XII
c	1,15	1,10	1,5	1,0	1,5	1,10	1,15	1,20	1,25	1,30	1,25	1,20
1 cubit	1,15	1,10	1,5	1,0	1,5	1,10	1,15	1,20	1,25	1,30	1,25	1,20
2 cubit	37;30	35	32.30	30	32.30	35	37;30	40	42;30	45	42;30	40
3 cubit	25	23;20	21;40	20	21;40	23;20	25	26;40	28;20	30	28;20	26;40
4 cubit	18;45	17;30	16;15	15	16;15	17;30	18;45	20	21;15	22;30	21;15	20
5 cubit	15	14	13	12	13	14	15	16	17	18	17	16
6 cubit	12;30	11;40	10;50	10	10;50	11;40	12;30	13;20	14;10	15	14;10	13;20
8 cubit	9;22,30	8;45	8;7,30	7;30	8;7,30	8;45	9;22,30	10	10;37,30	11;15	10;37,30	10
9 cubit	8;20	7;46,40	7;13,20	6;40	7;13,20	7;46,40	8;20	8;53,20	9;26,40	10	9;26,40	8;53,20
10 cubit	7;30	7	6;30	6	6;30	7	7;30	8	8;30	9	8;30	8

Table 2: Reconstruction of the complete MUL.APIN shadow length scheme. The value c corresponds to the constant value of the product of the shadow length and the time after sunrise for the given month. Numbers are written here in sexagesimal place value notation.

The mathematical nature of the shadow-length data in MUL.APIN has been noted already by van der Waerden (1951: 34), Neugebauer (1975: 544–545), and others. In addition to the two mathematical rules which form the basis of the scheme, its mathematical character is apparent from the fact that an entry for a 1 cubit shadow at the winter solstice is included. According to the scheme, the shadow will be 1 cubit in length at 1,30 UŠ (= 90 UŠ) after sunrise at the winter solstice. However, it is assumed in MUL.APIN that the length of daylight at the winter solstice is equal 2,0 UŠ (= 120 US); thus, 1,30 UŠ after sunrise would put us in the afternoon when the shadow is starting to get longer again. As the length of daylight at the winter solstice is 2,0 UŠ, the time 1,30 UŠ after sunrise is equal to 30 UŠ before sunset on that day. Since the changing length of the shadow is symmetrical around midday, the shadow length at 30 UŠ before sunset is equal to that at 30 UŠ after sunrise, and so according to this scheme for the winter solstice the shadow will be 3 cubits in length. At noon, the shadow will reach its shortest length of 1 1/2 cubits. The inclusion of the entry for 1 cubit can therefore only be explained by a desire to fill out the mathematical scheme to 1 cubit in parallel with the entries for the other months.

It is worth noting that despite its purely mathematical nature, the MUL.APIN shadow length scheme is on the whole a reasonable approximation to reality (Steele 2013: 9–10). However, it is not a scheme that would be very useful in practice. Rather than give the length of shadow at given time intervals during the day, the scheme does the opposite: it gives the time after sunrise when the shadow reaches lengths given in 1 cubit intervals (with the exception of 7 cubits). However, as can easily be seen in table 2, the shadow length changes very quickly near sunrise but very slowly close to midday. For example, at the summer solstice, it takes 30 UŠ (= 2 hours) for the shadow length to increase from 1 cubit to 2 cubits, but only 0;40 UŠ (= 2.66 minutes) to increase from 9 cubits to 10 cubits. Indeed, the time interval between a shadow of 1 cubit and of 2 cubits is always longer than that between 2 cubits and 10 cubits. Thus, the shadow length changes too slowly near midday and too quickly shortly after sunrise or before sunset to be useful for determining the time. Furthermore, the scheme as presented is backwards to what we would expect for time measurement: it starts with the shortest shadow, which is close to noon, the gives data for shadow lengths that corresponds to times progressively earlier in the day.

How should we understand this scheme, therefore? Neugebauer noted a similarity between the scheme and Old Babylonian reciprocal tables, writing that "The arithmetical structure of this table reflects the arrangement of the Old Babylonian tables of reciprocals – note the omission of the "irregular" number $s = 7$" (Neugebauer 1975: 544). It should be remarked, however, that what Neugebauer is comparing with the reciprocal tables is something similar to my table 2, in which the time after sunrise is given as a sexagesimal number. Whilst there is a mathematical relationship between the scheme and reciprocals, there is no direct similarity between the passages given in MUL.APIN, which are written as prose statements rather than in tabular form and which present quantities with units, and reciprocal tables, which operate purely with sexagesimal numbers. Thus, despite its mathematical nature, the shadow-length scheme in MUL.APIN does not read like a mathematical text. Rather, as is the case elsewhere in MUL.APIN and in other texts of schematic astronomy, simple mathematical tools have been applied to creating a numerical scheme representing the variation of an astronomical quantity, and the scheme is presented using the format and language of astronomical rather than mathematical texts.

4 Shadow-length schemes in Late Babylonian texts

I know of four tablets from the Late Babylonian period which expand upon or otherwise refer to the shadow-length scheme presented in MUL.APIN. Two of these texts are clearly astronomical. The first, BM 29371 (Brown, Fermor and Walker 1999/2000: 144–148; Hunger 1999: 134–135; Steele 2013: 28–32), was copied by Nabû-apla-iddin of the Ešguzi-mansum family, who was active in Borsippa during the mid to late 6th century BC (Waerzeggers 2012: 296). It contains an expansion of the MUL.APIN scheme to present the length of the shadow at 1 2/3 *bēru* after sunrise for every fifth day in the schematic calendar. This length of shadow follows a simple zigzag function with minimum 1;12 cubits and maximum 1;48 cubits, which is in agreement with the underlying principles of the MUL.APIN scheme. The layout of the tablet emphasises the fact that both the length of daylight and the behaviour of the shadow are symmetrical about the summer solstice: each line of the tablet gives two sets of identical data for the weight of water in a waterclock corresponding to the length of day and the length of the shadow at 1 2/3 *bēru* after sunrise for two dates centred on the summer solstice (Steele 2013: 28–32). The second tablet, BM 33564 (Steele 2013: 32–36), is a fragment of an astronomical procedure text. It was written by a member of the Mušēzib family (almost certainly Marduk-šapik-zēri, who was active in the early third century BC (Oelsner 2000)). Unfortunately, the tablet is badly damaged and its contents cannot be fully understood, but enough is preserved to see that the topic of the first section is the change in the length of the shadow and the length of daylight between the summer and winter solstices, and the second section deals with the length of daylight and the motion of the sun through the zodiac. The layout, terminology, and content of both texts put them firmly within the genre of astronomy.

The two remaining texts, BM 35369+45721 and SpTU IV 172, however, do not fit so clearly within the context of astronomy. Indeed, the shadow-length material on SpTU IV 172 appears at the end of a metrological table, and, as I will discuss, the style and language of BM 35369+45721 displays more similarities with mathematical problem texts than with astronomical texts. Let us consider each text in detail.

4.1 BM 35369+45721

BM 35369+45721 is a substantial fragment of a cuneiform tablet. The tablet, which is most likely from Babylon, cannot be dated any more precisely than to the Late Babylonian period. In Steele (2013: 14–26) I edited and discussed BM 45721[9]. However, my discussion was marred by confusion over the reading of the signs ḪÉ.EN as ḪÉ.GÁL[10]. In addition, during a visit to the British Museum in December 2019 I identified BM 35369 as part of the same tablet[11]. I therefore present here a new edition of the tablet with the new join before discussing its content below.

Obv.

1' [...] x [...]

2' ⌜DIŠ *ina* $^{\text{iti}}$APIN⌝ *šá* 1 DANNA 10 ⌜UŠ u_4-*mu*⌝ [ḪÉ.EN GIŠ.MI ...]

3' DIŠ *ina* $^{\text{iti}}$GAN *šá* 1 DANNA 12 UŠ 30 ⌜u_4-*mu*⌝ Ḫ[É.EN GIŠ.MI ...]

4' 42,30 A-RÁ 24 DU-*ma* 17 1,25 A-R[Á ...]

5' DIŠ *ina* $^{\text{iti}}$AB *šá* 1 DANNA ḪÉ.EN GIŠ.MI IGI$^?$ 2$^?$ 5 [...]

6' DIŠ *ina* $^{\text{iti}}$ŠU *šá* 4 DANNA u_4-*mu* ḪÉ.EN GIŠ.MI 1 ⌜*ina* 1 NIM-*ma*⌝ [...]

7' ⌜DIŠ⌝ *ina* $^{\text{iti}}$IZI *šá* 3 2/3 DANNA u_4-*mu* ḪÉ.EN GIŠ.MI 50 *ina* 1,5 ⌜NIM-*ma* 15 ZAL⌝ [3 SI GIŠ.MI ...]

8' [DIŠ] *ina* $^{\text{iti}}$KIN *šá* 2/3 (error for: 3) 1/3 DANNA u_4-*mu* ḪÉ.EN GIŠ.MI 40 ⌜*ina* 1,10 NIM-*ma*⌝ 30 ZAL 6 [SI GIŠ.MI ...]

[9] My discussion was subsequently summarized and discussed by Friberg and Al-Rawi (2016: 121).

[10] The excursus on p. 20 of Steele (2013) discussing the possible meaning of ḪÉ.GÁL, where I wrongly dismiss the more likely reading ḪÉ.EN on the basis of it being an unattested logographic combination, can now be ignored. I thank Matthew Rutz for first pointing out to me the occasional use of ḪÉ.EN in Late Babylonian mathematical texts.

[11] Unfortunately, I was not able to check the physical join at that time. On 16 January 2021 Jeanette Fincke informed me that she had independently identified the likely join of the same two tablets from photographs and I thank her for informing me of this. In November 2021 I was able to confirm the physical join of the two pieces at the British Museum.

9' [DIŠ] *ina* ^{iti}DU$_6$ *šá* 3 DANNA u_4-*mu* ḪÉ.EN GIŠ.MI 30 *ina* 1,15 NIM-*ma* 45 ZAL ⌜9⌝ [SI GIŠ.MI . . .]

10' [DIŠ *ina* ^{it}]^{i}APIN 2 2/3 DANNA u_4-*mu* ḪÉ.EN GIŠ.MI 20 *ina* 1,20 NIM-*ma* 1 ZAL ⌜1 KÙŠ GIŠ⌝.[MI . . .]

11' [DIŠ *ina* ^{i}]^{ti}GAN 2 1/3 DANNA u_4-*mu* ḪÉ.EN GIŠ.MI 10 *ina* 1,25 NIM-*ma* 1,15 ZAL 1 KÙŠ 3 [SI GIŠ.MI . . .]

12' [DIŠ *ina*] ^{iti}AB 2 DANNA u_4-*mu* ḪÉ.EN GIŠ.MI 1,30 KÙŠ GIM KA 1 KÙŠ 6 SI GIŠ.M[I . . .]

13' [DIŠ *ina* ^{iti}ŠU U]D-15-KAM 1 KÙŠ GIŠ.MI 2 DANNA u_4-*mu* 21 1 KÙŠ 1 ŠE GIŠ.MI 2 KASKAL UD.27 ⌜1 KÙŠ 2⌝ [ŠE]

14' [DIŠ *ina* ^{iti}IZI] ⌜UD.3⌝.KAM 1 KÙŠ 3 ŠE GIŠ.MI 2 DANNA u_4-*mu* [UD.9.K]AM 1 KÙŠ 4 ⌜ŠE GIŠ⌝.MI

15' [2 DANNA u_4]-*mu* UD.15.KAM 1 KÙŠ 1 SI GIŠ.MI 2 DA[N]NA UD.21.KAM 1 KÙŠ 1 ⌜SI$^{?}$ ⌝ 1 ŠE

16' [2] DANNA u_4-mu UD.⌜27.KAM⌝ 1 KÙŠ 1 S[I] ⌜2⌝ ŠE GIŠ.MI 2 DANNA ⌜u_4-mu⌝

17' [DIŠ *ina* ^{iti}KIN UD.3.KAM 1] ⌜KÙŠ⌝ 2 SI 3 Š[E 2 DANN]A ⌜u_4-*mu*⌝ UD.9.KAM 1 KÙŠ 1 ⌜SI⌝ 4 ŠE GIŠ.MI

18' [2 DANNA u_4-*mu* UD.15.KAM 1 KÙŠ 2 SI GIŠ.MI 2 DAN]NA u_4-mu [UD.21.KAM] ⌜x⌝

19' [. . .] ⌜x x⌝ [x x]

Rev.

1 [DIŠ *ina* ^{iti}DU$_6$ UD.3.KAM 1 KÙŠ 2 SI 3 ŠE GIŠ.MI 2 DANNA u_4]-*mu* UD.9.[KAM 1 KÙŠ 2 SI 4 ŠE]

2 [GIŠ.MI 2 DANNA u_4-*mu* . . .] ⌜GIŠ⌝.MI 2 KASKAL.BU u_4-*mu*

3 [. . .] x [. . . UD.27].KAM 1 KÙŠ 3 SI 2 ŠE GIŠ.MI ⌜x⌝

4 [DIŠ *ina* ^{iti}APIN UD.3.KAM 1] KÙŠ 3$^{?}$ SI 3 ŠE GIŠ.M[I 2 DANNA] u_4-*mu* UD.9 1 KÙŠ 3 SI 4 ŠE

5 [GIŠ.MI 2 DANNA u_4]- ⌜*mu*⌝ UD.15.KAM 1 KÙŠ 4 SI GIŠ.M[I 2] DANNA u_4-*mu* UD.21.KAM

6 [1 KÙŠ 4 SI 1 ŠE GIŠ].MI 2 DANNA u_4-*mu* UD.27.KAM 1 KÙŠ 4 SI ⌜2⌝ ŠE GIŠ.MI 2 KASKAL

7 [DIŠ *ina* $^{\text{iti}}$GAN UD.3.KAM 1 K]ÙŠ 4 SI 3 ŠE 2 DANNA u_4-*mu* UD.9.KAM [1 KÙŠ 4 SI 4 ŠE GIŠ.MI]

8 [2 DANNA u_4-*mu* U]D.15.KAM 1 KÙŠ 5 SI 2 DANNA u_4-*mu* UD.21.KAM [1 KÙŠ 5 SI 1 ŠE GIŠ.MI]

9 2 DANNA u_4-*mu* UD].27.KAM 1 KÙŠ 5$^?$ SI 2 ŠE GIŠ.MI ⌜2 DANNA⌝ [u_4-*mu*]

10 [DIŠ *ina* $^{\text{iti}}$AB UD.3.KAM 1] KÙŠ 5 SI 3 ŠE GIŠ.MI 2 DANNA [u_4-*mu* UD.9.KAM 1 KÙŠ 5 SI 4 ŠE GIŠ.MI]

11 [2 DANNA u_4]-*mu* UD.15.KAM 1 KÙŠ 6 SI x GIŠ.[MI . . .]

12 [2 DANNA u_4]-*mu* UD.⌜27.KAM⌝ [. . .]

13 [. . . x]+6$^?$ ŠE$^?$ GIM$^?$ UD.27$^?$.KAM UD-10[+x-KAM . . .]

14 [. . .] UD.15.KAM UD.21.KAM x UD.9.KA[M . . .]

15 [. . .] GIM $^{\text{iti}}$GAN$^?$ UD.27.KAM UD.10[+x.KAM . . .]

16 [. . .] UD.21.KAM GIM UD.9.KA[M . . .]

17 [. . .] GIM $^{\text{iti}}$DU$_6$$^?$ UD.27.[KAM . . .]

18 [. . . U]D.21.KAM GIM UD.9.[KAM. . .]

19 [. . .] x [. . .]

Obv.

1' [. . .] ... [. . .]

2' ¶ In Month VIII at 1 *bēru* 10 UŠ of day [What is the shadow? . . .]

3' ¶ In Month IX at 1 *bēru* 12 UŠ 30 (NINDA) of day. Wh[at is the shadow? . . .]

4' 42;30 multiplied by 24 is 17. 1,25 multip[lied by...]

5' ¶ In Month X at 1 *bēru*. What is the shadow? ... 2,5 [...]

6' ¶ In Month IV at 4 *bēru* of day. What is the shadow? 1. Subtract it from 1 and [...]

7' ¶ In Month V at 3 2/3 *bēru* of day. What is the shadow? 0;50 Subtract it from 1;5 and 0;15 is delayed [3 fingers of shadow ...]

8' [¶] In Month VI at 3 1/3 *bēru* of day. What is the shadow? 0;40. Subtract it from 1;10 and 0;30 is delayed 6 [fingers of shadow ...]

9' [¶] In Month VII at 3 *bēru* of day. What is the shadow? 0;30. Subtract it from 1;15 and 0;45 is delayed 9 [fingers of shadow ...]

10' [¶ In] Month VIII, 2 2/3 *bēru* of day. What is the shadow? 0;20. Subtract it from 1;20 and 1;0 is delayed 1 cubit of shadow [...]

11' [¶ In] Month IX, 2 1/3 *bēru* of day. What is the shadow? 0;10. Subtract it from 1;25 is 1;15 is delayed 1 cubit 3 [fingers of shadow ...]

12' [¶ In] Month X, 2 *bēru* of day. What is the shadow? 1;30 cubits which corresponds to 1 cubit 6 fingers of shad[ow ...]

13' [¶ In Month IV,] 15th [d]ay 1 cubit of shadow at 2 *bēru* of day. 21<st day> 1 cubit 1 barleycorn of shadow at 2 *bēru*. 27(th) day 1 cubit 2 barleycorn.

14' [¶ In Month V,] 3rd day 1 cubit 3 barleycorn of shadow at 2 *bēru* of day. [9th da]y 1 cubit barleycorn of shadow

15' [at 2 *bēru* of d]ay. 15th day 1 cubit 1 finger of shadow at 2 *bē*[*r*]*u* of shadow. 21st day 1 cubit 1 finger 1 barleycorn

16' [of shadow at] 2 *bēru* of day. 27th day 1 cubit 1 fin[ger] 2 barleycorn of shadow at 2 *bēru* of day.

17' [¶ In Month VI, 3rd day 1] cubit 2 fingers 2 barley[corn of shadow at 2 *bē*]*ru* of day. 9<th> day 1 cubit 1 finger 4 barleycorn of shadow

18' [at 2 *bēru* of day. 15th day 1 cubit 2 fingers of shadow at 2 *bē*]*ru* of day. [21st day]

19' [...] ... [...]

Rev.

1 [¶ In Month VII, 3rd day 1 cubit 2 fingers 3 barleycorn of shadow at 2 *bēru* of d]ay. 9[th] day [1 cubit 2 fingers 4 barleycorn]

2 [of shadow at 2 *bēru* of day. 15th day 1 cubit 3 fingers] of shadow at 2 *bēru* of day

3 [. . .] ... [. . . 27]th [day] 1 cubit 3 fingers of shadow . . .

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4 [¶ In Month VIII, 3rd day, 1] cubit 3 fingers 3 barleycorn of shad[ow at 2 *bēru*] of day. 9<th> day 1 cubit 3 fingers 4 barleycorn

5 [of shadow at 2 *bēru* of d]ay. 15th day 1 cubit 4 fingers of shad[ow 2] *bēru* of day. 21st day

6 [1 cubit 4 fingers 1 barleycorn of sha]dow at 2 *bēru* of day. 27th day 1 cubit 4 fingers 2 barleycorn of shadow at 2 *bēru*.

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7 [¶ In Month IX, 3rd day, 1 cu]bit 4 fingers 3 barleycorn at 2 *bēru* of day. 9th day [1 cubit 4 fingers 4 barleycorn of shadow]

8 [at 2 *bēru* of day.] 15th [d]ay 1 cubit 5 fingers at 2 *bēru* of day. 21st day [1 cubit 5 fingers 1 barleycorn of shadow]

9 [at 2 *bēru* of day.] 27th [day] 1 cubit 5 fingers 2 barleycorn of shadow at 2 *bēru* [of day]

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10 [¶ In Month X, 3rd day, 1] cubit 5 fingers 3 barleycorn of shadow at 2 *bēru* [of day. 9th day 1 cubit 5 fingers 4 barleycorn of shadow]

11 [at 2 *bēru* of d]ay. 15th day 1 cubit 6 finger . . . sha[dow . . .]

12 [2 *bēru* of d]ay. 27th day [. . .]

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13 [. . .] . . . Month XII$^{?}$ corresponds to 27?th day [xth] day [. . .]

14 [. . .] 15th day 21st day . . . 9th day [. . .]

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15 [. . .] corresponds to Month IX$^{?}$ 27th day [xth] day [. . .]

16 [. . .] 21st day corresponds to 9th day [. . .]

17 [. . .] corresponds to Month VII 27[th] day [. . .]

18 [. . .] 21st [d]ay corresponds to 9th day [. . .]

19 [. . .] . . . [. . .]

The tablet can be understood as containing four parts: Obv. 1'–5' (beginning of section lost), Obv. 6'–12', Obv. 13' – Rev. 12, and Rev. 13–18 (end of section lost). Each part is divided into several subsections by horizontal rulings (note that no distinction is made between the rulings marking the division into the four larger parts and those marking the subsections on the tablet). When complete, the first three sections each contained seven subsections. Each of these subsections concerns one month in the calendar, beginning with Month IV and ending with Month X. In the schematic calendar, the summer solstice is placed on the 15th of Month IV and the winter solstice is on the 15th of Month X. Thus, within each of these three parts of the text we run through the months from the summer solstice to the winter solstice. The fourth and final part explains that the situation is symmetrical for the other half of the year.

The first part, Obv. 1'–5', is badly preserved but enough remains to determine that we are asked to find the length of the shadow at given moments after sunrise. The entries for Month VIII and IX specify that this time is 1 *bēru* 10 UŠ and 1 *bēru* 12 UŠ 30 NINDA respectively. According to the MUL.APIN shadow length scheme, both of these cases correspond to a shadow length of 2 cubits. The entry for Month X gives the time as 1 *bēru*, which would correspond to a shadow length of 3 cubits. It seems possible that the scribe has made a mistake here and the time should have been 1 1/2 *bēru* (a simple error of omitting the sign for 1/2), which would also correspond to a 2 cubit-length shadow. Unfortunately, the procedure for calculating the shadow is damaged in each case. However, we can speculate that it was something along the following lines. Recall that in MUL.APIN, the time after sunrise multiplied by the length of the shadow is equal to a constant (this constant is simply the time after sunrise when the shadow equals 1 cubit according to the scheme). Thus, the shadow length can be found from the time by dividing that constant by the time. In Mesopotamia, division is typically carried out by taking the reciprocal of the divider and then multiplying the result by the dividend. We seem to have a hint of this at the end of the preserved part of Obv. 5', concerning Month X, where we read the sign IGI, which can indicate that reciprocal of a number is to be found. Thus, the very simple procedure of taking a reciprocal and then multiplying by the constant provides the required shadow length. This simple procedure breaks down, however, in the case of Month IX. The time after sunrise 1 *bēru* 12 UŠ 30 NINDA is 42;30 UŠ and 42;30 is not a regular sexagesimal number. Thus, we cannot simply take the reciprocal of the time. Instead, either an approximation to the reciprocal must be calculated, or else the length of the shadow must be determined in a different way. This likely explains the fact that the entry for Month IX takes up two lines on the tablet whereas those for Month VII and X fit on one line.

In the second part of the text, Obv. 6'–12', we are again asked to find the shadow from a stated time after sunrise. In this case, this time corresponds to the moment of midday. The shadow is found by subtracting a number b from a number c to give a result d (all in sexagesimal place value notation). This result is named ZAL, almost certainly coming from the Akkadian verb *uḫḫuru* "to be late" which can be written using the logogram ZAL, and is then converted into a quality in cubits and fingers where 1 cubit = 12 fingers (a metrology which is specific to this text; see further the discussion of part 3 below). The numbers b and c can be understood as the time of midday after sunrise minus 60 UŠ and the time after sunrise when the shadow reaches 1 cubit in length respectively (Steele 2013: 22). The resulting shadow length differs from that implied by the MUL.APIN scheme (indeed, it is considerably superior to it – see Steele (2013: 23)), but I cannot explain the rationale behind the computation or what is meant by the term "is delayed"[12].

4.2 SpTU IV 172

The tablet SpTU IV 172 (W 23273), which was excavated from the so-called 'house of the āšipus' from Achaemenid Uruk, contains a copy of a list of numbers associated with gods, a metrological table, and a short section on shadows (Friberg and Al-Rawi 2016: 106–124; Proust 2019: 100–106). The majority of the tablet is taken up with the metrological table. This table presents metrologies for length (both rod and cubit metrologies are given), area and weight. The two length metrologies are each presented twice, first as sexagesimal numbers followed by length quantities with units, and then in the opposite order of length quantities with units followed by sexagesimal numbers. Immediately after the metrological tables is a catch-line for a metrological table of capacities on another tablet. Below this, we find the material on shadows, which implies that it was added to this tablet after the copying of the metrological tables. Finally, the tablet ends with a colophon

[12]Friberg and Al-Rawi (2016: 121) dismiss this section as containing "nonsense calculations and (an) incorrect counting of fingers".

which explains that the tablet was owned by Rīmūt-Anu, son of Šamaš-iddin, descendent of Šangî-Ninurta. Rīmūt-Anu was the owner of a number of scholarly cuneiform tablets found in this house, including medical texts, omen texts, and another mathematical tablet (Robson 2008: 232; Proust 2019: 94–96).

The shadow-length material copied at the end of the tablet was probably divided into two sections, although damage to the middle of this material has destroyed the horizontal ruling marking the section boundaries. The text reads as follows[13]:

39 [. . .] *am-mat* ŠU

40 [. . .] KÙŠ IZI *u* SIG$_4$ KI.MIN

41 [. . . KÙŠ KIN] *u* GU$_4$ KI.MIN

42 [. . . KÙŠ DU$_6$ *u*] ⌜ BÁR ⌝ KI.MIN

43 [. . . KÙŠ APIN *u* ŠE KI].MIN

44 [. . . KÙŠ GAN *u* ZÍZ KI.MIN]

45 [. . . KÙŠ AB]

---

46 [ŠU . . . GIŠ.MI ZAL-*ra*]

47 [IZI 15] ⌜GIŠ⌝.MI ZAL-*ra*

48 KIN 30 GIŠ.MI ZAL-*ra*

49 DU$_6$ 45 GIŠ.MI ZAL-*ra*

50 APIN 1 ⌜GIŠ⌝.MI ZAL-*ra*

51 GAN 1,15 GIŠ.MI ZAL-*ra*

52 AB 1,30 GIŠ.MI ZAL-*ra*

[13] SpTU IV 172 Rev. IX 39–52. The edition given here has one extra line in the break between lines 43 and 46 than is shown in the copy published in SpTU IV or in my previous edition published (Steele 2013: 26). The presence of this extra line, which is expected from the content of the text, seems plausible based upon the photograph published by Friberg and Al-Rawi (2016: 124), and is included in their edition without comment.

39 [. . .] cubit Month IV.

40 [. . .] cubits Month V and Month III the same.

41 [. . . cubits Month VI] and Month II the same.

42 [. . . cubits Month VII and] Month I the same.

43 [. . . cubits Month VIII and Month XII the sa]me.

44 [. . . cubits Month IX and Month XI the same]

45 [. . . cubits Month X]

46 [Month IV . . . shadow is delayed.]

47 [Month V 15] shadow is delayed.

48 Month VI 30 shadow is delayed.

49 Month VII 45 shadow is delayed.

50 Month VIII 1 shadow is delayed.

51 Month IX 1,15 shadow is delayed.

52 Month X 1,30 shadow is delayed.

The first section apparently contained statements of the length of the shadow for different months of the year. In accord with MUL.APIN and all of the other shadow length schemes, the scheme is symmetrical around the solstices in Months IV and X. Unfortunately, the numbers at the beginnings of each line are lost. Two possibilities suggest themselves for restoring these numbers. First, the numbers could be the length of the shadow at 60 UŠ (= 2 *bēru*) after sunrise, following the MUL.APIN scheme, which follow a simple zigzag function from 1 cubit to 1;30 cubits. This number is also the constant c in the relationship between the shadow length and the time after sunrise in the MUL.APIN scheme (Steele 2013: 27). Alternatively, the numbers could duplicate those in the second section, thus giving two versions of the same material, similar to the sections of the metrological text dealing with length metrologies earlier in the tablet (Friberg and Al-Rawi 2016: 120). The second section presents the same numbers calculated in part 2 of BM 35369+45721, again designated by the obscure phrase "is delayed". These numbers seem to be alternative lengths for the shadow at noon to those implied by the MUL.APIN scheme (Steele 2013: 27–28).

Whatever the meaning and purpose of these shadow-length material on this tablet, what is important for the present discussion is that this material is found on a mathematical tablet presenting metrological tables, not an astronomical tablet. SpTU IV 172, therefore, provides clear evidence of an astronomical topic, the variation in the length of shadow over the course of the year, in an explicitly mathematical context.

5 Shadow-length schemes: between mathematics and astronomy

Although the variation in the length of the shadow cast by a gnomon is essentially an astronomical problem, and a simple scheme representing this variation is presented in MUL.APIN, probably the most widely copied and read astronomical text within Mesopotamia, BM 35369+45721 and SpTU IV 172 show that during the Late Babylonian period, this astronomical problem was also incorporated into mathematical texts. The scheme presented in MUL.APIN is based upon two simple mathematical foundations: the assumption that on any given date the length of the shadow multiplied by the time after sunrise is equal to a constant, and that this constant, and therefore also both the time after sunrise when the shadow reaches a given length and length of the shadow at a given time, vary as simple zigzag functions. The simplicity and the basic mathematical nature of these foundations of the scheme may well have made it an ideal topic for mathematical investigation.

The context of scholarship during the Late Babylonian period – a relatively small number of scholars, employed by the temples, and who were active in many areas of learning – provided a suitable environment for the interaction of different types of scholarship which had until then remained largely separate. In particular, astronomical and astrological ideas were incorporated into a wide range of other scholarly areas, such as medicine, liver divination, ritual, and historical writing (Heeßel 2008; Geller 2011, 2014; Krul 2019, Reynolds 2019). Given this context, we should not be surprised that astronomy was also incorporated within mathematical scholarship. On the contrary, what is perhaps surprising is that we do not have more cases of astronomy providing the setting for mathematical problems or for mathematical texts which investigate the mathematical properties of functions employed within astronomy.

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